October 8, 2009

AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION

JR/INTER SCHOOL DIVISION

SOLUTIONS, CONTEST # 1 2009-2010

Adviser: Score the contest using the answers below or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

ANSWERS

1. T represents 8
R represents 7
Y represents 9
M represents 6
A represents 2
H represents 4

$$\frac{T - R + Y}{M + A + T \div H} = \frac{8 - 7 + 9}{6 + 2 + 8 \div 4}$$

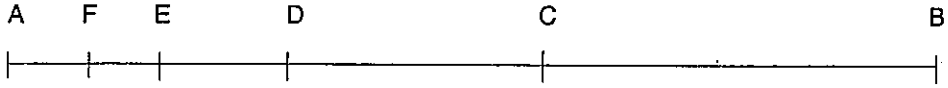
by order of operations:

$$\frac{1 + 9}{6 + 2 + 2} = \frac{10}{10} = 1$$

ANSWER: 1

1. 1

2.



If $\overline{AF} = 3$, $\overline{AE} = 6$, $\overline{AD} = 12$, $\overline{AC} = 24$ and $\overline{AB} = 48$ ANSWER: 48 or 48 units

2. 48 units

3. Let p = number of plates bought

$$(2/3)p + (1/2)p - (1/4)p + 2 = p$$
$$(8/12)p + (6/12)p - (3/12)p + 2 = p$$
$$(11/12)p + 2 = (12/12)p$$
$$2 = p/12$$
$$24 = p$$

ANSWER: He bought 24 plates

3. 24 plates

4. 44 mph for 198 lasted 4 and 1/2 hours
300 - 198 = 102 miles to go in 2 hours
You must average 51 mph to arrive on time. ANSWER: 51 mph

4. 51 mph

5.

		Omit these because 2 adjacent "off"'s:	
ON-ON-ON-ON	OFF-ON-OFF-ON	ON-ON-OFF-OFF	OFF-OFF-OFF-ON
ON-OFF-ON-ON	OFF-ON-ON-ON	ON-OFF-OFF-OFF	OFF-OFF-ON-OFF
ON-OFF-ON-OFF	OFF-ON-ON-OFF	ON-OFF-OFF-ON	OFF-ON-OFF-OFF
ON-ON-ON-OFF		OFF-OFF-OFF-OFF	
ON-ON-OFF-ON		OFF-OFF-ON-ON	

ANSWER: 8

5. 8

6. Let x = first odd integer and x + 2 = second odd integer

$$x + x + 2 + x(x + 2) = 119$$
$$2x + 2 + x^2 + 2x = 119$$
$$x^2 + 4x + 2 = 119$$
$$x^2 + 4x - 117 = 0$$
$$(x + 13)(x - 9) = 0$$
$$x + 13 = 0 \qquad x - 9 = 0$$
$$x = -13 \qquad x = 9$$
reject $x + 2 = 11$

ANSWER: 9, 11 in any order

6. 9, 11 in any order

7. Let c = regular price of the cartridge
c - 15% of c = 0.85c
0.85c - 0.10(0.85c) = 25.09
0.85c - 0.085c = 25.09
0.765c = 25.09
c = 32.80 (32.797 ≈ 32.80)

ANSWER: $32.80

7. $32.80

November 12, 2009

AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION

JR/INTER SCHOOL DIVISION

SOLUTIONS, CONTEST # 2 2009-2010

Adviser: Score the contest using the answers below or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

ANSWERS

1. Let xy = original 2 digit number. Let xy3 = new 3-digit number
xy + 372 = xy3, y must be 1
x1 + 372 = x13, x must be 4
41 + 372 = 413, xy must be 41 ANSWER: 41

1. 41

2.

Mr. Ties	B	G	W
B	X	√	X
G	X	X	√
W	√	X	X

Mr. Shirts	B	G	W
B	X	X	√
G	√	X	X
W	X	√	X

Mr. White is wearing a blue shirt

ANSWER: Blue

2. Blue

3. Work backwards: Let x = number
60% more than 10 is 10 + 6 = 16
25% less than 16 is 16 - 4 = 12
50% more than x = 12
.5x + x = 12
1.5x = 12
x = 8 ANSWER: 8

3. 8

4. There are six possible outcomes:
ABC; ACB; BAC; BCA; CAB; CBA
There is only 1 possible outcome in alphabetical order. Therefore, the probability of the horses finishing a race in alphabetical order is $\frac{1}{6}$. ANSWER: $\frac{1}{6}$

4. $\frac{1}{6}$

5. Since 35 = 5 x 7 and 77 = 7 x 11,
(5 x 11) = 55 must also be a factor.
The factors we can list are:
1, 5, 7, 11, 35, 55, 77 and 5 x 77 or 385. ANSWER: 385

5. 385

6. Let x = the number $\frac{1}{2}x + \frac{2}{3}x = 42$

$$\frac{3x}{6} + \frac{4x}{6} = 42$$

$$\frac{7x}{6} = 42$$

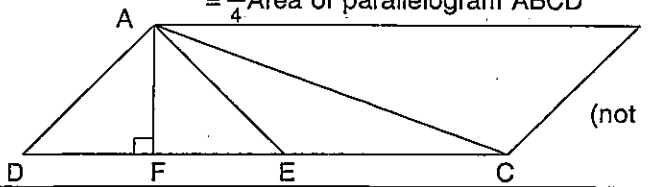
$$7x = 252$$

$$x = 36$$

ANSWER: 36

6. 36

7. Parallelogram ABCD has height AF. Triangle ADE has height AF
Area Δ ADE = $\frac{1}{2}$AF • DE. Area of parallelogram ABCD = AF • 2DE
Area Δ ADE = $\frac{1}{2}$ AF • $\frac{1}{2}$2DC = AF • DC
= $\frac{1}{4}$ (AF • DC)
= $\frac{1}{4}$ Area of parallelogram ABCD



(not drawn to scale)

ANSWER: 1:4 or $\frac{1}{4}$

7. 1:4 or $\frac{1}{4}$

December 10, 2009

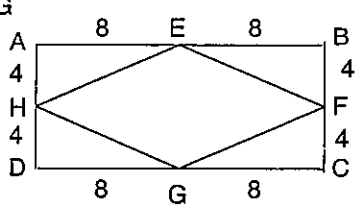
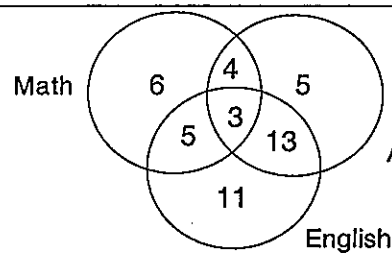
AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION

JR/INTER SCHOOL DIVISION

SOLUTIONS, CONTEST # 3 2009-2010

Adviser: Score the contest using the answers below or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

ANSWERS

1. To write the numerals 1, 2, 3, . . . , 100, the following are used: 11 zeros, 21 ones and 20 each of twos, threes, . . . nines for a total of 192 digits. Their sum is 901. Thus the mean is 901/192; median is 5, mode is 1. ANSWER: Mean = 901/192 or 4.6875, Median = 5, Mode = 1 (all answers required)	1. Mean = 901/192 or 4.6875 Median = 5 Mode = 1 (all answers required)
2. Work backwards. The third man left 8 apples, which is 2/3 of what he found. 8 = 2/3 of x. x = 12. He found 12 apples. The second man left 12 apples, which is 2/3 of what he found. 12 = 2/3 of y. y = 18. He found 18 apples. The first man left 18 apples, which is 2/3 of what he found. 18 = 2/3 of z. z = 27. ANSWER: 27 apples	2. 27 apples
3. Let l = length, w = width l = 2w $P_{ABCD} = 48$ 2l + 2w = 48 2(2w) + 2w = 48 6w = 48 w = 8, l = 16 $A_{ABCD} = lw = 16 \cdot 8 = 128$ $\triangle AEH \cong \triangle EBF \cong \triangle FCG \cong \triangle HDG$ $A_{EFGH} = A_{ABCD} - 4A_{\triangle AEH}$ $A_{EFGH} = 128 - 4\left(\frac{8 \cdot 4}{2}\right)$ $= 128 - 64$ $A_{EFGH} = 64$ square units  ANSWER: 64 sq. units	3. 64 sq. units
4.  Adding, 5 + 4 + 3 + 13 + 6 + 5 + 11 = 47 ANSWER: 47	4. 47
5. 12 dozen oranges at 24¢ a dozen = $2.88 spent 12 x 12 = 144 - 18 bad oranges = 126 sold at 3/8¢ = $3.36 earned. $3.36 - $2.88 = $0.48 gain .48/2.88 = x/100 2.88x = 48 x = 16 2/3 % ANSWER: 16 2/3% gain	5. $16\frac{2}{3}$% gain
6. 9 notebooks cost either $9.55, 9.60, 9.65, 9.70, 9.75, 9.80, 9.85, or 9.90. Using the rules of divisibility, the only number that works is: 9 notebooks for $9.90 or 1 notebook for $1.10. ANSWER: $1.10	6. $1.10
7. Let F = Fredburger, m = milk shake, f = order of fries. (1) F + m + f = $3.72, (2) F = m + 2f, (3) 3m = F + f (1) and (3): m + 3m = $3.72, m = $0.93 (3) 3m = F + f, F = 3($0.93) - f, F = $2.79 - f (2) m = F - 2f, $0.93 = ($2.79 - f) - 2f, $1.86 = 3f, f = $0.62 (2) F = $0.93 + 2($0.62), F = $2.17 Fredburger = $2.17, milkshake = $0.93, 1 order of fries = $0.62 ANSWER: $2.17 + $0.93 + $0.62 + $0.62 = $4.34	7. $4.34

January 14, 2010

AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION

JR/INTER SCHOOL DIVISION

SOLUTIONS, CONTEST # 4 2009-2010

Adviser: Score the contest using the answers below or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

ANSWERS

1. 96 wagons - 12 broken down = 84 left.
Each took 1 more person, therefore the 12 broken wagons had 84 people in all or
$84 \div 12 = 7$ people in each originally ANSWER: 7

1. 7

2. $3*[(7\#3)\#(3*4)] = 3*[(7\#3)\#(3\cdot 4 + 1)]$
$= 3*[(7\#3)\#13]$
$= 3*\left[\left(\frac{7+3}{2}\right)\#13\right]$
$= 3*[5\#13]$
$= 3*\left[\frac{5+13}{2}\right]$
$= 3*9$
$= 3 \cdot 9 + 1 = 28$ ANSWER:28

2. 28

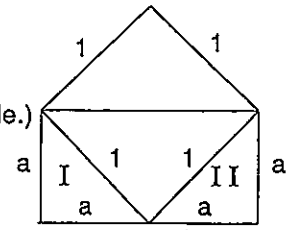
3. Let x = amount larger barrel holds.
5/6 of 336 = 280 L
280 = 4/9 of larger barrel
$280 = 4/9 \cdot x$
$2520 = 4x$, $630 = x$ The larger barrel holds 630 L ANSWER: 630 L

3. 630 liters

4. Area square = 1 x 1 = 1, Area $\Delta I = \frac{b \times h}{2} = \frac{a^2}{2}$
But, by Pythagorean Theorem,

$a^2 + a^2 = 1$
$2a^2 = 1$
$a^2 = 1/2$ therefore $\frac{\frac{1}{2}}{2} = \frac{1}{4} = \text{Area}\Delta I$
$\text{Area}\Delta II = 1/4$
Area of shape = $1 + \frac{1}{4} + \frac{1}{4} = 1\frac{1}{2}$

(Not drawn to scale.)



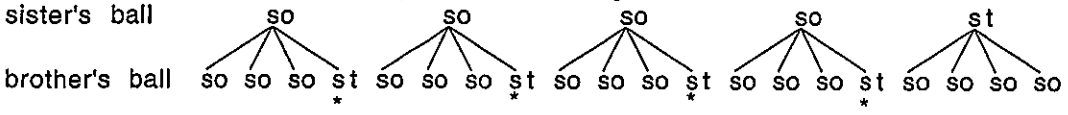
ANSWER: $1\frac{1}{2}$

4. $1\frac{1}{2}$

5. 5% of \$1,500 = \$75 commission
7% of \$1,500 = \$105 commission
10% of \$12,800 = \$1,280 commission
Total sales = \$15,800 Total commission = \$1,460 ANSWER: \$1,460

5. \$1,460

6. Let so = solid and st = striped The following trees show all the outcomes:



The favorable outcomes are starred. Out of the 20 outcomes,
4 show brother's ball to be striped. ANSWER: 4/20 or 1/5 or 0.2

6. 4/20 or 1/5 or 0.2

7. Larry has 89 dimes = \$8.90
112 quarters = \$28.00
7 half-dollars = \$3.50
67 one-dollar bills = \$67.00
7 five-dollar bills = \$35.00
total = \$142.40
\$144.00 - \$142.40 = \$1.60. This means he needs 32 nickels,
but since he has MORE THAN ENOUGH, he must have 33 nickels. ANSWER: 33

7. 33 nickels

February 11, 2010

AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION

JR/INTER SCHOOL DIVISION

SOLUTIONS, CONTEST #5 2009-2010

Adviser: Score the contest using the answers below or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

ANSWERS

1. In half a day the clock chimes: $12(4 + 8 + 12 + 16) + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12$ $= 480 + 78 = 558$ chimes. In one day, the clock chimes $558 \times 2 = 1116$ In one week, the clock chimes $1116 \times 7 = 7812$ times ANSWER: 7812	1. 7812
2. Area ABCD = $25^2 = 625$ sq. cm 625 sq. cm - (25% of 625) = 625 - 156.25 Area EFGH = $156.25 = (12.5)^2$ Dimensions of sq. EFGH - one side = 12.5 cm ANSWER: 12.5 or 12 1/2 cm	2. 12.5 or 12 1/2 cm
3. $\frac{5280 \text{ feet (in a mile)}}{1056 \text{ revolutions (in a mile)}} = 5$ feet (in a revolution) Therefore circumference of wheel is 5 $2\pi r = 5$ feet $r = \frac{5}{2\pi}$ feet, $r \approx 0.8$ feet ANSWER: About 0.8 feet	3. About 0.8 feet
4. $(\frac{1}{2})^{-1} = 2$ $[(\frac{1}{2})^{-1} + (\frac{1}{3})^{-1} + (\frac{1}{5})^{-1} + (\frac{1}{7})^{-1}]^{-1}$ $(\frac{1}{3})^{-1} = 3$ $[2 + 3 + 5 + 7]^{-1}$ $(\frac{1}{5})^{-1} = 5$ $(17)^{-1} = \frac{1}{17}$ $(\frac{1}{7})^{-1} = 7$ ANSWER: $\frac{1}{17}$	4. $\frac{1}{17}$
5. 14190 2 x 7095 5 x 1419 3 x 473 11 x 43 Make a factor tree to find prime factors 43 - 11 = 32 ANSWER: 32	5. 32
6. Let f = number of females, Let m = number of males $f/m = 7/4$; $4f = 7m$; $f = 7/4m$ $(f - 3)/(m - 12) = 5/2$; $2(f - 3) = 5(m - 12)$; $2f - 6 = 5m - 60$ by substituting, $2(7/4m) - 6 = 5m - 60$; $7/2m - 6 = 5m - 60$; $54 = 3/2m$; $36 = m$ $f = 7/4(36) = 63$ $(f - 3) = 63 - 3 = 60$ $(m - 12) = 36 - 12 = 24$. $60 + 24 = 84$ ANSWER: 84	6. 84
7. Ed 18,750 Monica 12,500/46,250 = 10/37 of the deli Dana 15,000 Monica 12,500 10/37 of 18,500 = 5000 ANSWER: $5000 Total 46,250	7. $5000

March 11, 2010

AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION

JR/INTER SCHOOL DIVISION

SOLUTIONS, CONTEST # 6 2009-2010

Adviser: Score the contest using the answers below or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

ANSWERS

1. Because ABC is isosceles, $\overline{CD}$ bisects $\overline{AB}$ so that $\overline{AD} = 15$. We can find $\overline{CD}$:

$(\overline{AD})^2 + (\overline{CD})^2 = (\overline{AC})^2$

$15^2 + (\overline{CD})^2 = 39^2$

$225 + (\overline{CD})^2 = 1521$

$(\overline{CD})^2 = 1296$

$\overline{CD} = \sqrt{1296} = 36$

From a differenct perspective.

$\text{Area}\Delta ABC = 1/2bh$

$540 = 1/2 \cdot 39 \cdot h$

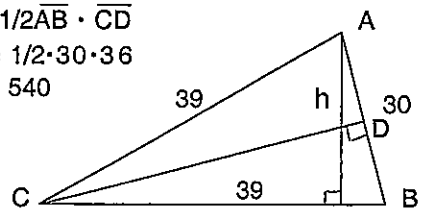
$1080 = 39 \cdot h, \quad h \approx 27.7$

$\text{Area}\Delta ABC = 1/2bh$

$= 1/2\overline{AB} \cdot \overline{CD}$

$= 1/2 \cdot 30 \cdot 36$

$= 540$



ANSWER: ≈ 27.7

1. about 27.7

2. Let x = number sold

Day	Number to be sold	Number sold	Number left
Mon.	10	x	10 - x
Tues.	2(10 - x) = 20 - 2x	x	(20 - 2x) - x = 20 - 3x
Wed.	3(20 - 3x) = 60 - 9x	x	(60 - 9x) - x = 60 - 10x

$60 - 10x = 0$

$60 = 10x, \quad 6 = x$ ANSWER: 6

2. 6

3. Let d = original price: let p = profit

$p = 0.2d$

$$p + 20 = \frac{d + 20}{3}$$

$3p + 60 = d + 20$

$3(0.2d) + 60 = d + 20$

$0.6d + 60 = d + 20$

$40 = .4d$

$100 = d$ ANSWER: $100

3. $100

4. Let t = Dr. McKay's age now: $3/13t + 4/39t + 1/2t + 13 = t$

$18/78t + 8/78t + 39/78t + 13 = t$

$65/78t + 13 = t$

$13 = 13/78t$

$1014 = 13t, \quad 78 = t$ ANSWER: 78 years old

4. 78 years old

5. By looking: 1:11 is the time when the fewest lights are lit.

10:08 is the time when the greatest number of lights are lit.

ANSWER: 1:11 and 10:08 (both answers required)

5. 1:11 and 10:08 (both answers required)

6. Use each guess to find a common sum or difference:

Karen's guess: $59 + 16 = 75$, $59 - 16 = 43$, $59 + 19 = 78^*$

$59 - 19 = 40$, $59 + 43 = 102$, $59 - 43 = 16$

Beth's guess: $94 + 16 = 110$, $94 - 16 = 78^*$, $94 + 19 = 113$

$94 - 19 = 75$, $94 + 43 = 137$, $94 - 43 = 57$

Jarrett's guess: $121 + 16 = 137$, $121 - 16 = 105$, $121 + 19 = 140$

$121 - 19 = 102$, $121 + 43 = 164$, $121 - 43 = 78^*$ ANSWER: 78 pounds

6. 78 pounds

7. Let x = number of freshmen from neither WHS or LHS.

Since all of WHS graduating students go to Augusta, 640 go to Augusta + 1/2 of 960 of LHS graduating students or 480 go to Augusta. But 1/2 of Augusta's freshmen class is from WHS so 1280 students are in Augusta's freshman class.

$1280 = 480 + 640 + x$

$1280 = 1120 + x$

$160 = x$ ANSWER: 160

7. 160

Note: Since this is the last contest of the year, the adviser must make sure the cumulative score for each student is indicated on the scorecard so that we can issue the award for your highest-scoring student.